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Effect of looking at the car that follows in an optimal velocity model of traffic flow

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An extension of an optimal velocity model is proposed. In the new model, a driver looks at the following car as well as the preceding car. We introduce an additional optimal velocity function that depends on the headway of the following car. We investigate the effect of looking back at the car that follows and show that this extension effectively stabilizes the traffic flow.

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I. INTRODUCTION

The investigation of traffic flow, especially of traffic congestion, is an interesting problem from a physical viewpoint. Figure 1 shows some observed data of the freeway traffic, the so-called fundamental diagram, which expresses the relation between the flow and the density of cars [1]. The traffic flow is divided into two states, a free flow state (the left-hand side of the peak) and a congested flow state (the right-hand side). If the density is low, one can drive freely and the flow is almost proportional to the density. While the density becomes high, drivers are forced to reduce the velocity and congestion emerges.

There have been many attempts at constructing models for freeway traffic to explain the mechanism of congestion from the viewpoint of physics [2–4]. In recent years, cellular automaton models [5,6], coupled map models [7], and fluid dynamical models [8] have successfully described the dynamical formation of traffic congestion. The optimal velocity (OV) model is a kind of car following model, which is very simple and has succeeded in showing the dynamical formation of congestion [9–14]. In the OV model, the change of traffic flow can be understood as a kind of phase transition. If the car density exceeds the critical value, the traffic flow becomes unstable and the congestion appears dynamically. The behavior of each car is described by a solitonlike solution of the equation of the OV model. Moreover, simulations based on this model reproduce well the real data in the fundamental diagram. The diamond marks in Fig. 1 show the result of simulation. The global shape of figure is in good agreement with the observed flow. The transition point from the free flow state to the congested flow state coincides with the observed data. We note that there exists a metastable state in the vicinity of the critical density, which is an important property for characterizing the transition in real traffic. The metastable state is naturally induced in the result of the dynamical effect of the OV model [13,14]. The results of simulation by the coupled map model based on the OV model [15] in realistic situations, such as two lanes and junctions, are in good agreement with the real traffic data [16].

The basic property of the formation mechanism of congestions is most simply understood by the OV model. The OV model can be extended by taking into account the effect of other cars as well as the preceding car. It is quite natural to take account of the motion of the car next to the preceding car or the following car, as we often pay attention to the motion of such cars. In granular flow theories, each particle interacts with many other particles around it. In the unified viewpoint of granular and traffic theories, it may be meaningful to incorporate such an effect [17–21].

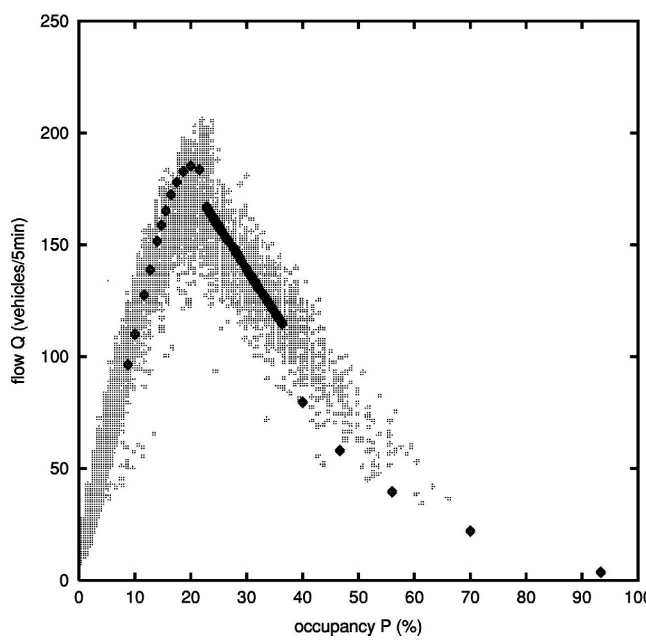

FIG. 1. Fundamental diagram: Small dots represent the observational data. Diamond marks are the results of simulations in the OV model.

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As is well known, the traffic congestion is an important problem not only from a physical viewpoint but also from a social viewpoint. If the traffic flow is in the congested state, much noise and much carbon dioxide are generated and much energy is wasted. In recent years, automatic driving control systems are utilized as a part of the intelligent transport system (ITS). The suppression of the appearance of traffic congestion is one of the target of the ITS. We can discuss how to stabilize the traffic flow in the context of the OV model [17,19], because the formation mechanism of traffic congestion is naturally described by the OV model. In this work we will attempt the most effective extension to the OV model. The new term, which incorporates the effect of looking back at the car that follows, is added to the original OV model, and the effect of this term will be investigated concerning the stabilization of traffic flow.

II. OV MODEL

First, we briefly review the original OV model and how the model explains the emergence of traffic congestions [9,12]. The model is formulated as

$$\frac{d^2x_n}{dt^2}=a\left[V(x_{n+1}-x_n)-\frac{dx_n}{dt}\right], \tag{1}$$

for each car number n ($n=1,2,\ldots$). x_n is the position of the nth car and $x_{n+1}-x_n$ is the headway of this car. a is a constant called "sensitivity," which we set at the same value for all drivers. $V(x)$ is called the optimal velocity function (OV function), which expresses the relation between headway and the optimal velocity of each car. A driver controls acceleration or deceleration according to the difference between the optimal velocity and his own velocity.

The OV function has the following property: the function becomes zero for a small headway and approaches the maximum value for sufficiently large headway. The typical example of the OV function is

$$V(x)=\alpha[\tanh(x-\beta)+\gamma], \tag{2}$$

where α, β, and γ are some positive constants.

The OV model has a homogeneous flow solution as

$$x_n^{(0)}(t)=bn+V(b)t, \tag{3}$$

where all cars are uniformly distributed, $x_{n+1}^{(0)}-x_n^{(0)}=b$, and are moving with the same velocity $V(b)$. The homogeneous flow can be identified as the free flow in real traffic. We can examine the stability of this solution by adding a small perturbation [9]. We put $x_n(t)=x_n^{(0)}(t)+y_n(t)$ and linearize with respect to $y_n(t)$. $y_n(t)$ can be expanded by Fourier modes $\exp[ikn+i\omega(k)t]$. The condition where the homogeneous flow solution is stable for a perturbation is given by $\mathrm{Im}\,\omega(k)>0$ for all k. The result is

$$a>2V'(b)=\frac{2\alpha}{\cosh^2(b-\beta)}, \tag{4}$$

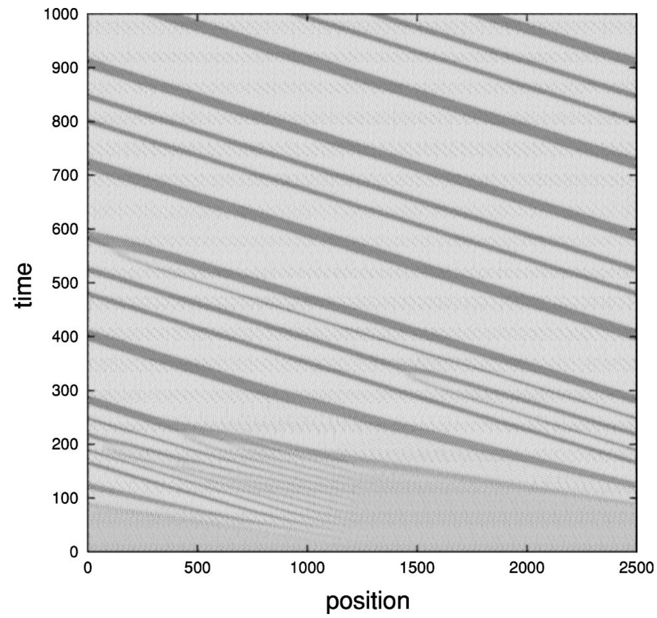

FIG. 2. A typical pattern of congestion formation in the circuit: The initial condition of the simulation is taken as the homogeneous flow. The positions of the cars are plotted. Cars are moving from left to right, and congestion clusters are moving backward in the direction of the cars.

where V' is a derivative of the OV function (2). The stability of the homogeneous flow is decided by sensitivity a and mean headway b, which is the inverse of the car density. If a is smaller than 2α, there exists a critical density for given a. In the unstable case, the congestion is formed with time evolution. A typical example of the congestion formation is shown in Fig. 2.

III. EXTENDED OV MODEL

Now, we investigate extended models to suppress the formation of congestion. In the OV model, the appearance of congestion can be suppressed by choosing high sensitivity. Here we show that it is not the best way and present another possibility, an extended model. In our extended model, a driver looks at the following car as well as the preceding car. We call it the backward looking OV (BL-OV) model. Nagatani has investigated a different extended model from ours for the same purpose. His model incorporates the next-nearest-neighbor interaction [19], where a driver looks at the preceding and the next to the preceding cars.

The BL-OV model is presented by the equation

$$\frac{d^2x_n}{dt^2}=a\left[\{V_F(x_{n+1}-x_n)+V_B(x_n-x_{n-1})\}-\frac{dx_n}{dt}\right], \tag{5}$$

where $V_F(x)$ is the OV function for forward looking that plays the same role as $V(x)$ in Eq. (1). $V_B(x)$ is the OV function for backward looking, which is a function of the headway of the following car [17].

We choose two OV functions as

$$V_F(x)=\alpha'[\tanh(x-\beta)+\gamma], \tag{6}$$

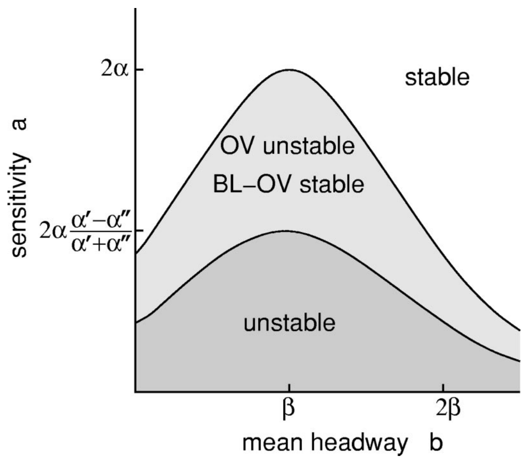

FIG. 3. The phase diagram of the OV and BL-OV models: In the upper region, the homogeneous flow is stable for both models. In the middle region it is unstable only for the OV model, and in the lower region it is unstable for both models.

$$V_B(x) = -\alpha''[\tanh(x-\beta)+\gamma],$$

where α', α'', β, and γ are positive constants. Though β (γ) may take different values for V_F and V_B, we set the same value as β (γ) in the original OV model for simplicity. The velocity of the car is controlled according to the OV functions, depending on both the distance to the preceding car and to the following car. The function $V_B(x)$ has the effect of increasing the velocity of the car, if the headway of the following car becomes small. In this model, each car is controlled so as to be positioned at the middle point between the preceding car and the following car.

In the same way as the OV model, we can find the homogeneous flow solution: $x_{n+1}-x_n=b$ and $\dot{x}_n=V_F(b)+V_B(b)$. The stability condition is

$$a > 2\frac{[V_F'(b)+V_B'(b)]^2}{V_F'(b)-V_B'(b)}, \tag{7}$$

which reduces to Eq. (4) when we switch off V_B. To compare the OV and BL-OV models, we assume that they have the same homogeneous flow solution (3) for any mean headway b. This condition is equivalent to $V(b)=V_F(b)+V_B(b)$, which results in $\alpha=\alpha'-\alpha''$. Using this relation together with Eq. (6), Eq. (7) is rewritten as

$$a > \frac{2\alpha}{\cosh^2(b-\beta)}\left(\frac{\alpha'-\alpha''}{\alpha'+\alpha''}\right). \tag{8}$$

Obviously, the quantity on the right-hand side in Eq. (8) is always smaller than that in Eq. (4) for the original OV model. This means the free flow becomes stable, even if we take low sensitivity where the flow is unstable in the original OV model. The phase diagram of the original and extended OV models clearly shows this result (Fig. 3). The stable area for the BL-OV model is larger than that for the OV model.

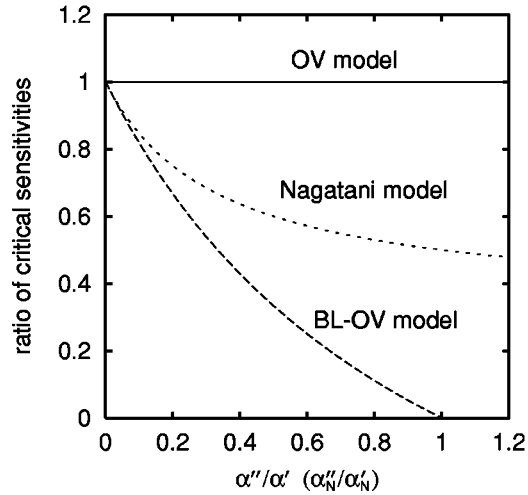

FIG. 4. The critical sensitivities of extended OV models compared to the OV model (solid line) are shown. The dashed line is the ratio of critical sensitivities of the BL-OV to OV models, $(\alpha'-\alpha'')/(\alpha'+\alpha'')$. The dotted line is the ratio of critical sensitivities of the Nagatani model to the OV model, $(\alpha_N'+\alpha_N'')/(\alpha_N'+3\alpha_N'')$.

In a similar way, we can find the stability condition of another extended model, which incorporates the next-nearest-neighbor interaction:

$$\frac{d^2x_n}{dt^2} = a\left[\{V_F(x_{n+1}-x_n)+V_{FF}(x_{n+2}-x_{n+1})\}-\frac{dx_n}{dt}\right], \tag{9}$$

where V_F and V_{FF} are OV functions for the preceding car and the next to the preceding car, respectively. The stability condition is

$$a > 2\frac{[V_F'(b)+V_{FF}'(b)]^2}{V_F'(b)+3V_{FF}'(b)}. \tag{10}$$

If we use the explicit form $V_F=\alpha_N'[\tanh(x-\beta)+\gamma]$, $V_{FF}=\alpha_N''[\tanh(x-\beta)+\gamma]$ and $\alpha=\alpha_N'+\alpha_N''$;[1] then Eq. (10) becomes

$$a > \frac{2\alpha}{\cosh^2(b-\beta)}\left(\frac{\alpha_N'+\alpha_N''}{\alpha_N'+3\alpha_N''}\right). \tag{11}$$

The stability of the homogeneous flow state obviously increases in this model also. In order to show the relation among the stability conditions of these three models, we plot the ratio of critical sensitivities Eq. (8) to Eq. (4) as well as Eq. (11) to Eq. (4) (Fig. 4). These ratios can be written only by α''/α' or α_N''/α_N', which expresses the ratio of the new

[1]This condition comes from the assumption that the model has the same homogeneous flow solution (3) for any mean headway b. The original form of OV function in Ref. [19] is $(1-\gamma)V(x_{n+1}-x_n)+\gamma V(x_{n+2}-x_{n+1})$, and therefore the condition is automatically satisfied.

term to the original term of the OV functions.[2] The BL-OV model stabilizes the homogeneous flow more than the original OV model and another extended model incorporating the contribution of the next to the preceding car.

We also investigate the relaxation time of a small disturbance. $\mathrm{Im}\,\omega(k)$ can be evaluated in the same way as the derivation of Eq. (4). As mentioned in Sec. II, the stability condition of the homogeneous flow solution is $\mathrm{Im}\,\omega(k)>0$ for all k. The relaxation time is determined by the lowest value of $\mathrm{Im}\,\omega(k)$. We note that the unstability arises first from the longest wavelength mode $k\sim 0$. The lowest value of $\mathrm{Im}\,\omega(k)$ is obtained for $k=2\pi/N$, which is the minimum value of k, where N is the number of cars [9]. The lowest value of $\mathrm{Im}\,\omega(k)$ for the OV model is

$$\mathrm{Im}\,\omega \sim \frac{k^2}{2a}V'(b)[a-2V'(b)] = \frac{k^2}{2a}(a\alpha - 2\alpha^2), \quad (12)$$

and that for the BL-OV model is

$$\mathrm{Im}\,\omega \sim \frac{k^2}{2a}[V_F'(b)-V_B'(b)]\left\{a-2\frac{[V_F'(b)+V_B'(b)]^2}{V_F'(b)-V_B'(b)}\right\}$$
$$= \frac{k^2}{2a}[a(\alpha'+\alpha'')-2\alpha^2]. \quad (13)$$

Obviously the right-hand side of Eq. (13) is always larger than that of Eq. (12), as $\alpha=\alpha'-\alpha''$. Then the relaxation time for the BL-OV model is shorter than that for the OV model in any sensitivity and any choice of parameters of OV functions. The disturbance damps faster in the BL-OV model than in the OV model.

IV. SIMULATION

In order to demonstrate how the stability of free flow is improved in the BL-OV model, we perform the simulation. The situation is as follows: 100 cars are running on the circuit with the length 100. The mean headway is $b=1$ and the mean velocity is tanh(1). We take the parameters of OV functions as $\alpha=1.0$, $\beta=1$, $\gamma=\tanh(1)$, $\alpha'=1.3$, and $\alpha''=0.3$. The last two conditions come from the fact that two models must have the same homogeneous flow solution for any density. In this parameter setting, the homogeneous flow is stable under the conditions: $a>2$ for the OV model and $a>1.25$ for the BL-OV model.

First we show how fast the disturbance disappears. The initial condition is that only one car has larger headway than the others. The behavior in this situation is beyond linear analysis. Figure 5 shows the behavior of disturbance under the condition $a=2.5$. The disturbance is absorbed much faster in the BL-OV model than in the OV model. The stability of traffic flow is improved for the BL-OV model also when it exhibits nonlinear behavior.

From a social viewpoint, the consumption of fuel for driving cars is an important problem. Actually, traffic congestion results in the consumption of much fuel. Such quantity can be estimated by the changes of velocity, that is, the changes of the kinetic energy of cars. We compare the BL-OV and OV models with regard to "energy consumption." Suppose that a disturbance is added in the homogeneous flow, which propagates like a shockwave, then the velocity of each car

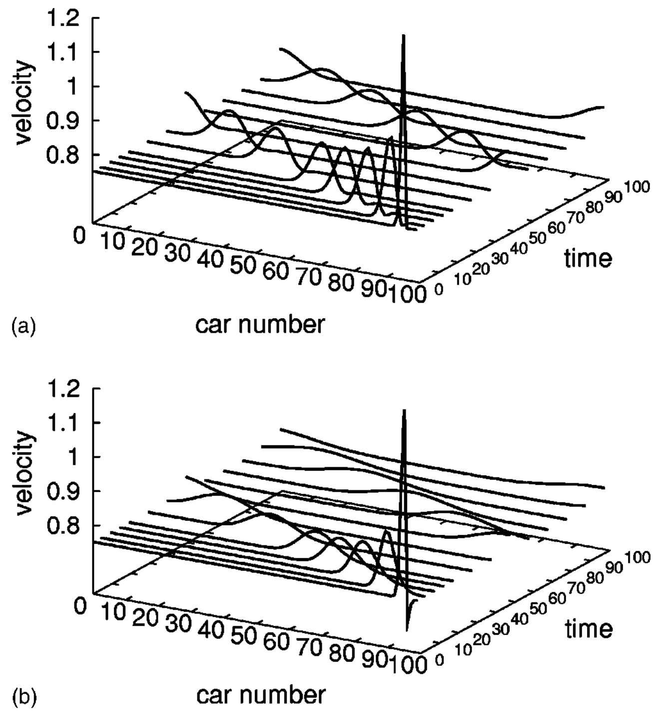

FIG. 5. Superposed solid lines represent the damping behavior of the disturbance for the original OV model (a) and for the BL-OV model (b). The headway of only one car is twice as long as that of the others in the initial condition. The shock wave travels in the left direction.

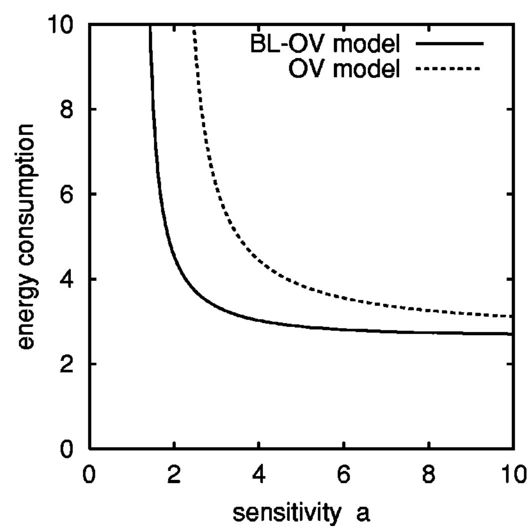

FIG. 6. The solid line represents the energy consumption for the BL-OV model and the dashed line represents that for the OV model. The energy consumption diverges at the critical sensitivity $a=2.0$ for the OV model and $a=1.25$ for the BL-OV model.

[2]In the BL-OV model, $\alpha''/\alpha'=1$ cannot be realized because of the condition $\alpha=\alpha'-\alpha''$.

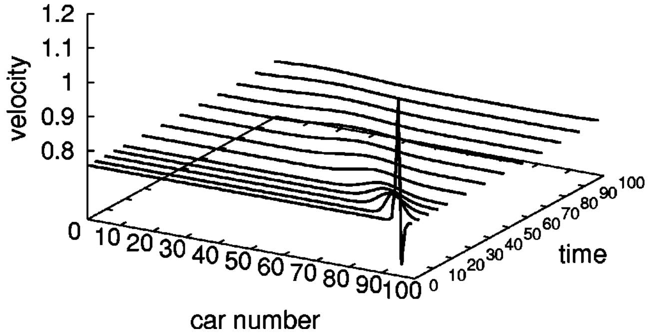

FIG. 7. Superposed solid lines represent the damping behavior of the disturbance for the BL-OV model with tuned parameters.

oscillates several times until the disturbance disappears (Fig. 5). The changes of velocity result in the additional consumption of energy compared to the case of no disturbance. We use the following quantity for estimating such additional energy consumption:

$$E=\sum_{\text{cars}}\sum_{\text{waves}}\frac{1}{2}(v_{max}^2-v_{min}^2), \tag{14}$$

where Σ_{wave} denotes the summation for all periods of oscillation until the disturbance disappears. Figure 6 shows the energy consumption for the BL-OV and OV models. The BL-OV model obviously causes less consumption of energy than the OV model in any value of sensitivity.

V. SUMMARY AND DISCUSSION

In this paper, we have investigated the effect of backward looking in the OV model. In our extended model, the stability of traffic flow has increased. This extension enables us to suppress the formation of congestion effectively and to reduce the energy consumption. The OV model and two extended models have been compared under the condition that they have the same homogeneous flow solution for any density. Among these models, the BL-OV model provides the most stable flow in any of the cases discussed in this paper.

The suppression of the emergence of congestion is one of the most important problems of the social domain. The BL-OV model has the ability to tune parameters for such a purpose. The parameters can be selected so as to stabilize the homogeneous flow state best at a given mean headway. For example, if we intend to stabilize the flow at $b=\beta=1$ and $\gamma=\tanh(1)$, we choose the OV function as

$$V_F(x)=0.7[\tanh(x-1)+\tanh(1)],$$

$$V_B(x)=0.3\ \ [-\tanh(x-1)+\tanh(1)]. \tag{15}$$

We note that the homogeneous flow solution under this condition is the same as that in the OV model (3) at the value $b=1$ only. In this parameter setting the flow becomes more stable. We demonstrate the improvement in the stability of flow by simulations (Fig. 7). The damping speed of disturbance is much faster than those in the previous section (Fig. 5). Figure 8 shows the energy consumption in this model.

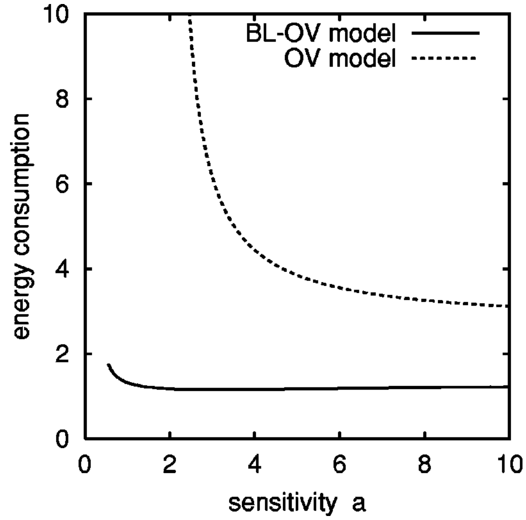

FIG. 8. The solid line represents the energy consumption for the BL-OV model with tuned parameters. The energy consumption for the BL-OV model diverges at the critical sensitivity $a=0.32$.

The choice of these parameters gives not only low energy consumption but also a small critical sensitivity. The above procedure of choosing parameters has a large advantage in stabilizing the free flow, when the BL-OV model is applied to the real traffic flow.

Through the investigation in this paper, we can suggest that any basic theory for the control of traffic flow should incorporate the effect of backward looking. This effect improves the stability of traffic flow concerned with the following two points. First, traffic flow is always disturbed by intersections or other road conditions in real traffic. So, how fast the flow absorbs such disturbances is important. If we take into account backward looking in the OV model, the disturbance in free flow damps much faster than the original model. Second, it is a general property that the formation of congestion is suppressed by developing high sensitivity. On the engineering side, high sensitivity requires that the control system respond sensitively to the change of headway and velocity. But it is technically difficult. As we have mentioned in Sec. III, the stability region is extended in the phase diagram for the BL-OV model (Fig. 3), which incorporates the effect of backward looking. This means that the effect stabilizes the traffic flow even at low sensitivity. The idea of backward looking is beyond the usual control of drivers, but can be realized by some engineering techniques of the ITS. Such a realization seems easier than the development of a sensitive control system.

ACKNOWLEDGMENTS

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